Walking with coffee: Why does it spill?

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In our busy lives, almost all of us have to walk with a cup of coffee. While often we spill the drink, this familiar phenomenon has never been explored systematically. Here we report on the results of an experimental study of the conditions under which coffee spills for various walking speeds and initial liquid levels in the cup. These observations are analyzed from the dynamical systems and fluid mechanics viewpoints as well as with the help of a model developed here. Particularities of the common cup sizes, the coffee properties, and the biomechanics of walking proved to be responsible for the spilling phenomenon. The studied problem represents an example of the interplay between the complex motion of a cup, due to the biomechanics of a walking individual, and the low-viscosity-liquid dynamics in it.

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I. INTRODUCTION

Abramson, a classic of sloshing dynamics, started his seminal 1967 NASA report [1] with the remark that "it is common everyday knowledge to each of us that any small container filled with liquid must be moved or carried very carefully to avoid spills." While many of us may have thought about this phenomenon, the question as to why we spill coffee has never been formally studied to the authors' knowledge. As we have all walked with a cup of coffee or other liquid-filled containers, this is certainly a familiar problem [see Fig. 1(a)].

The presented study of when and why coffee spills shows that it is a confluence of biomechanics, liquid sloshing engineering, and dynamical systems. While each of these subjects is well developed, the fact that they have different focuses of study does not allow one to penetrate into the posed question immediately. Namely, on one hand, a large body of the biomechanics and medical literature is dedicated to the study of *human walking*, including quantifying the motion of the body center of mass [2], energy expenditure, and efficiency of process [3]; gait patterns related to gender, age, health, etc. [2–5]; motion of appendages (arms and legs) in natural positions [6]; and regularity of walking over long periods of time [7]. On the other hand, *liquid sloshing* studies are concerned with large liquid-filled structures such as rocket fuel tanks [1,8,9], which are subject to considerable accelerations and vibrations; forces and torques due to liquids experiencing various sloshing motions [10] [see Figs. 1(b)–1(d)]; and suppression of sloshing necessary for vehicle control [11,12]. However, in the walking with coffee problem the motions of the human body, while seemingly regular, are quite complex and are coupled to a coffee cup and liquid therein, which makes it difficult to unravel the precise reasons behind coffee spilling.

II. OUR STUDIES

To address the challenge we set up experiments [13] in which a human subject was asked to walk naturally with different speeds along a straight line path [Fig. 2(a)], with acceleration $\ddot{x}$ chosen naturally by the subject. Two different regimes of walking were explored: when the subject was "focused"—trying not to spill and "unfocused"—not concerned with spilling [14]. The subject was recorded at 100 fps walking at different average speeds in the range $\langle\dot{x}\rangle = 0.5$–$2.5$ m/s. The motion was analyzed in the xy plane only [see Fig. 2(b)], which is justified below, using an image analysis program written in MATLAB, while the instant of a spill was determined with a light-emitting diode signal triggered by a sensor monitoring the coffee level in the cup [see Fig. 2(d)].

A. Cup excitation vs hydrodynamics in the cup

As known from biomechanics [3], the key goals during walking are to move the body toward a desired location and at a desired speed, while using the least amount of total energy (mechanical, muscular contraction, etc.) to achieve the first goal. This means that the body tends to move in as straight a line as possible and with very little up and down movements. The latter is due to the tendency at the freely chosen step rate to consume the least oxygen at any given speed [15,16]. As follows from the trajectory measurements [see Fig. 3(a)], an initial acceleration region is followed by a region of an almost constant walking speed, i.e., the maximum acceleration $\ddot{x}_{\max}$ is realized early on. Measurements, shown in Fig. 4(a), demonstrate that $\ddot{x}_{\max}$ chosen by a human subject increases with average speed $\langle\dot{x}\rangle$ and is lower in the focused regime for higher walking speeds. Since the maximum acceleration $\ddot{x}_{\max}$ is a good measure of the initial deflection of the liquid interface, we decided to choose it for further characterization of the data and scale it with respect to the acceleration $\ddot{x}_\alpha$, defined by the spill angle α for a given coffee level [see Fig. 2(c)]. Analyses of all cup excitations—back-and-forth in Fig. 3(a), vertical in Fig. 3(b), and pitching—show the presence of oscillatory-type motions on the order of 1 cm in magnitude superimposed on the average trajectory of the human subject. Vertical excitations are characterized by the maximum of the y velocity synchronized with steps [see Fig. 3(b)]. As for pitching θ motion (not shown), it is essentially a combination of arm swing and flexibility of the wrist and is on the order of 2°. Analysis of the dependence of excitation on walking speed in Fig. 4(c) indicates that the standard deviation of the x velocity about the average decreases with increasing walking speed. This is counterintuitive, i.e., the faster one walks the smoother the motion of the cup in the x direction, as

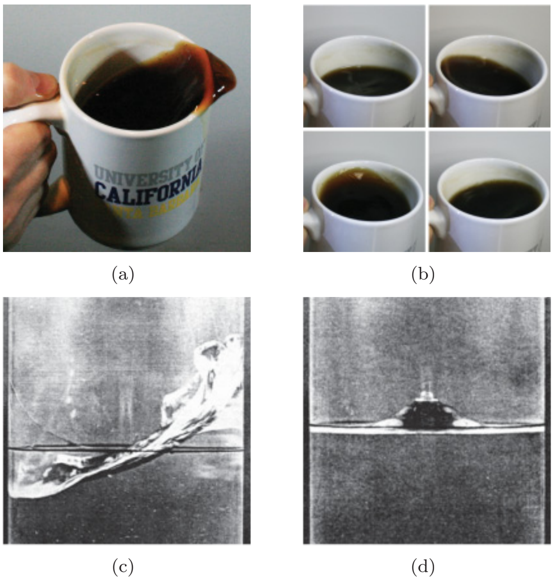

FIG. 1. (Color online) Coffee spill and key liquid motions in an excited cup. (a) Representative image of coffee spilling. (b) Rotational liquid motion in clockwise direction: top left–top right–bottom left–bottom right. (c) Back-and-forth liquid oscillations (photograph from [1]). (d) Vertical liquid oscillations (photograph from [1]).

opposed to the magnitude of the y excitation in Fig. 4(d), which increases with walking speed. Figure 4(b) shows that the coffee cup is excited at a frequency f_{cup} nearly identical to the step frequency f_{step}, which increases with walking speed and agrees with previous measurements for a variety of individuals [3]. Most importantly, the excitation frequency is in the range of $f_{\mathrm{cup}} = 1$–2.5 Hz.

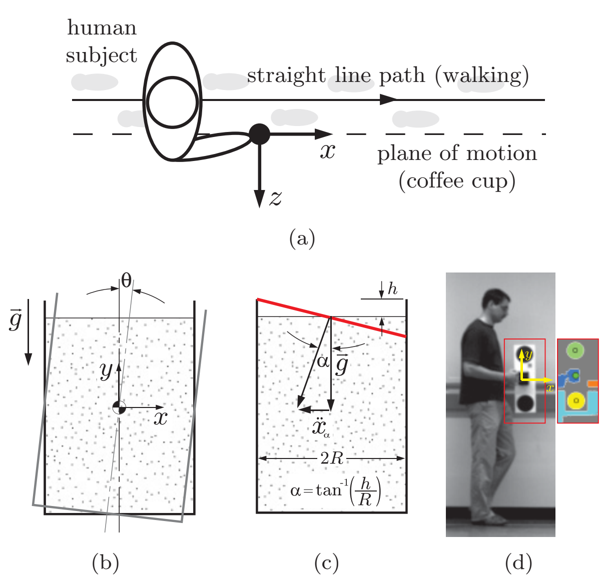

FIG. 2. (Color online) Definition and extraction of the cup dimensions and coordinates in the coffee spill experiments. (a) Walking path as viewed from above. (b) Cup coordinates: xy is the plane of motion, θ the pitching angle, and g gravity. (c) Spill angle α and equivalent acceleration $\ddot{x}_\alpha$. (d) MATLAB image analysis.

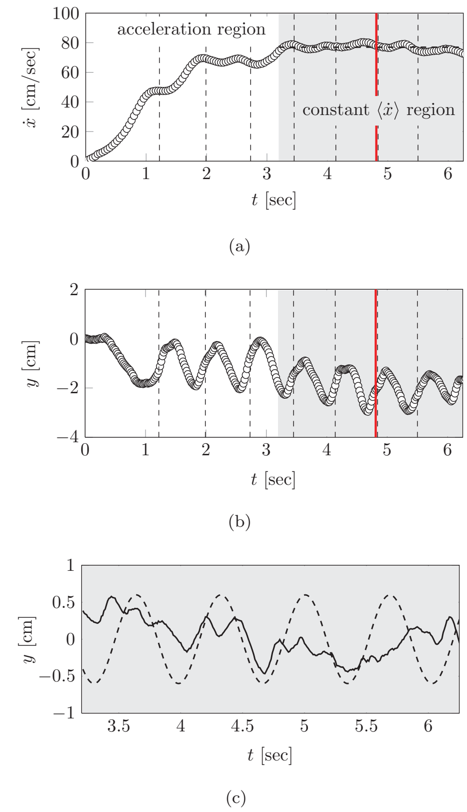

FIG. 3. (Color online) The trajectory of the coffee cup determined from image analysis for (a) back-and-forth and (b) vertical excitation. (c) The decomposition of the signal shown in the shaded region in (b) into smooth (dashed line) and noise (solid line) components. (a)–(c) depict cup excitations in the unfocused regime with walking speed $\langle \dot{x} \rangle = 80$ cm/s, maximum acceleration 71 cm/s, and coffee level $\alpha = 12.1^\circ$; shaded region corresponds to the regime of constant average velocity, the red (solid vertical) line shows the instant of the first coffee spill, and the dashed vertical lines correspond to step instants.

Now let us compare the cup excitations to natural frequencies of liquid oscillations in the cup. The natural frequencies of oscillations of a frictionless, vorticity-free, and incompressible liquid in an upright cylindrical container (cup) with a free liquid surface are well known from liquid sloshing engineering [17,18]:

$$\omega_{mn}^2 = \frac{g\,\epsilon_{mn}}{R}\tanh\left(\epsilon_{mn}\frac{H}{R}\right)\left[1+\frac{\sigma}{\rho g}\left(\frac{\epsilon_{mn}}{R}\right)^2\right], \tag{1}$$

where $m = 0,1,2,\ldots$ and $n = 1,2,\ldots$. In this equation H is the liquid height, ρ its density, R the cup radius, σ the surface tension, and g the gravity acceleration. The values ϵ_{mn} are the roots of the first derivative $J_m'(\epsilon) = 0$ of the mth-order Bessel function of the first kind; numerical values are $\epsilon_{11} = 1.841$, $\epsilon_{12} = 5.331$, $\epsilon_{31} = 8.536, \ldots, \epsilon_{1\,n+1} = \epsilon_{1\,n} + \pi$. The shape of an antisymmetric mode of the free surface wave is proportional to $J_1(\epsilon_{mn} r/R)$ and for $n = 1$ is shown in plate 191 of van Dyke's *Album of Fluid Motion* [19]. For a typical common size of a coffee cup, $R \simeq 3.5$ cm and $H \simeq 10$ cm, which is studied here, the surface tension σ effect is negligible [20] and the angular frequency is $\omega_{11} = \omega_{\mathrm{nat}} \sim 23$ rad/s, so

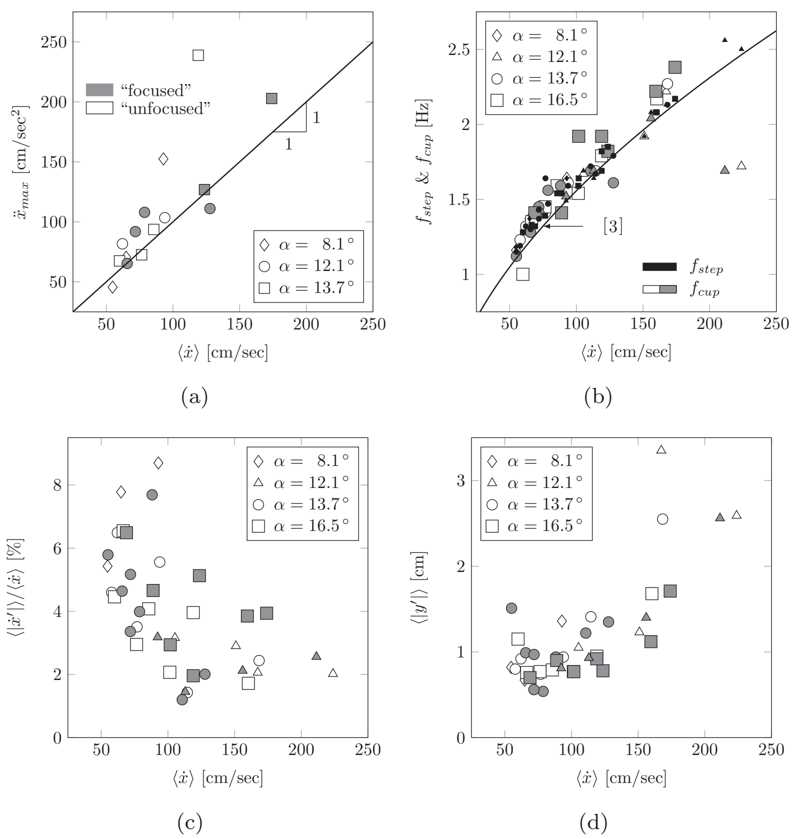

FIG. 4. (a) and (b) illustrate basic biomechanics for different coffee levels: (a) maximum acceleration $\ddot{x}_{\max}$ in focused and unfocused regimes for different walking speeds and (b) step (black symbols) and cup excitation frequencies and a comparison of our data to the empirical curve [3]. (c) and (d) show representative samples of the dependence of excitation on walking speed, specifically (c) the dependence of back-and-forth excitation $\langle \dot{x}' \rangle$ on walking speed $\langle \dot{x} \rangle$ and (d) the dependence of vertical excitation $\langle y' \rangle$ on walking speed $\langle \dot{x} \rangle$.

that the corresponding frequency is $\nu_{11} = \nu_{\text{nat}} \sim 3.65$ Hz and the period of liquid oscillations is $T_{\text{nat}} \sim 0.27$ s. In general, coffee cups vary in size roughly from $(R,H) = (2.5,5.7)$ to $(6.7,8.9)$ cm for espresso to cappuccino type mugs, which gives natural frequencies in the range 2.6–4.3 Hz. Therefore, once compared to the range of excitation frequencies, $f_{\text{cup}} =$ 1–2.5 Hz, it is the lowest-frequency antisymmetric mode ω_{nat} that tends to get excited during walking, which exhibits the largest liquid mass participating in the back-and-forth sloshing and thus leads to the largest effect on the dynamics of the system [see Fig. 1(c)].

From a dynamical systems point of view, the considered system is parametrically excited by the following motions:

(i) *Back-and-forth*: this excites the first (main) antisymmetric sloshing mode [see Fig. 1(c)].

(ii) *Lateral*: as known from the measurements of the motion of the body center of gravity during walking, the lateral and forward oscillations are of the same order of amplitude [2] of 20 mm, although the frequency of the lateral (in the xz plane) motion is almost half that of the forward motion [2], i.e., $\sim$1.25 Hz. Therefore, its effect is less important as it is further away from the natural frequency of liquid oscillations, $\nu_{\text{nat}} = 3.65$ Hz.

(iii) *Vertical*: Faraday [21] was one of the first to observe the phenomenon of parametric resonance, noting that surface waves in a fluid-filled cylinder under vertical excitation exhibited twice the period of the excitation—the so-called one-half subharmonic response. Since the frequency of vertical excitation in our case is lower than the natural one, one does not expect the Faraday instability to be observed [1,10].

(iv) *Pitching*: Since the pitching frequency is the same as that of back-and-forth motion, it contributes to the excitation of the natural antisymmetric mode [10].

These basic excitation effects alone, however, do not explain the particularities of coffee spilling. First, let us look at the dependence of the number of steps N before the coffee is spilled on the coffee level α, the walking speed characterized by the maximum acceleration $\ddot{x}_{\max}$ as per Fig. 4(a), and the focused vs unfocused regimes in Fig. 5. First, it was determined that most often coffee spills in the range of 4–5 m (7–10 steps) of walking distance. Clearly, N decreases with increasing $\ddot{x}_{\max}/\ddot{x}_{\alpha}$ and thus walking speed, which is consonant with one's intuition. If $\ddot{x}_{\max}/\ddot{x}_{\alpha} > 1$, a very short spill time is observed as the coffee reaches the critical spill angle α due to a large early acceleration that provides significant initial deflection of the interface (fluid statics condition); see Fig. 2(c). Also, high coffee levels (small α) exhibit fewer N before spilling occurs. Most interestingly, focusing tends to increase N before spilling, which, in part, is due to lower maximum acceleration, which sets a lower initial amplitude for liquid sloshing, and a lower noise level in the excitation for a given walking speed compared to the unfocused regime as explained below.

While walking appears to be a periodic, regular process, closer examination [7] reveals fluctuations in the gait pattern, even under steady conditions [22]. Together with other natural factors—uneven floors, distractions during walking, etc.—this explains why the cup motion during the constant walking speed regime is composed of noise and smooth oscillations of constant amplitude [see Fig. 3(c)]: the "smooth" signal has a frequency commensurate with that of walking, whereas it is apparent that the "noise" is a complicated combination of

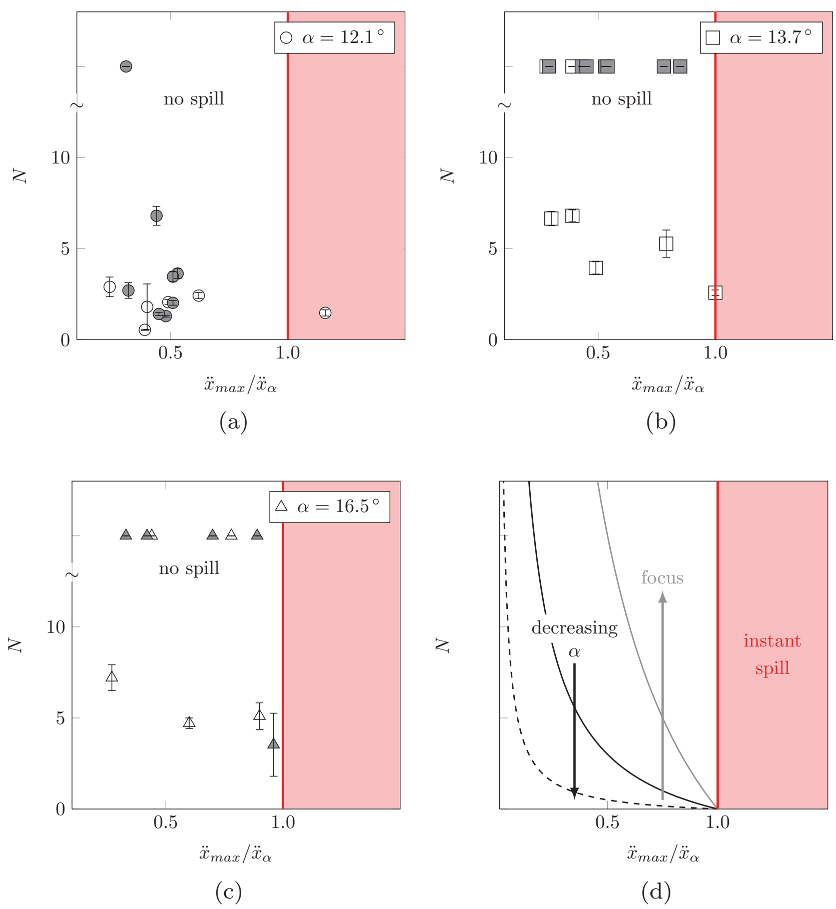

FIG. 5. (Color online) Experimental measurements of the number of steps N to spill coffee for different coffee levels α. (a)–(c) represent experimental measurements for different coffee levels α for both focused (shaded symbols) and unfocused (open symbols) regimes of walking, with summarized dependence on the coffee level α and focused vs unfocused regimes in (d); the error bars represent the uncertainty in determining the step time interval (used to transform spill time into steps to spill, N).

higher-frequency harmonics. Comparison of the focused and unfocused regimes in Fig. 5 suggests that higher levels of noise in the latter regime substantially intensify the spilling.

B. Mechanical model

To support the above conclusions and explain the experimental findings presented here, we developed a simple mechanical model based on the fully nonlinear spherical pendulum equations with forced oscillations [23]. As a spherical pendulum (along with spring-mass models [24]) have characteristics similar to sloshing, several investigators have used this analogy to predict and understand the observed sloshing behavior in other systems [8,25–27] .

Since we are mainly interested in the near-planar pendulum motions, it is convenient to choose a coordinate system which has singularities at $x = z = 0$, $y = \pm 1$ (the points on the sphere that are furthest from the plane) and is defined in terms of two angles γ and β:

$$x = \cos\beta\,\sin\gamma, \quad y = \sin\beta, \quad z = -\cos\beta\,\cos\gamma, \tag{2}$$

where $-\pi < \gamma \leqslant \pi$ and $-\pi/2 \leqslant \beta \leqslant \pi/2$ as shown in Fig. 6. Considering a spherical pendulum of length l and mass m, which is forced horizontally in the x direction as $x_0(t)$, the nondimensional kinetic and potential energies are given by

$$\frac{T}{m\,g\,l} = \frac{1}{2}[(\dot{x} + \dot{x}_0)^2 + \dot{y}^2 + \dot{z}^2], \quad \frac{V}{m\,g\,l} = z, \tag{3}$$

where we took into account that the horizontal displacements of the pendulum relative to the pivot are lx and ly, respectively. Given the expressions of the kinetic and potential energies, we can form the Lagrangian $L = T - V$ and, expressing x and z in terms of y and γ, derive the Euler-Lagrange equations [29] for γ and y:

$$\ddot{\gamma} - \frac{2\,y\,\dot{y}\,\dot{\gamma}}{1-y^2} + \frac{\sin\gamma}{\sqrt{1-y^2}} = -\frac{\ddot{x}_0\,\cos\gamma}{\sqrt{1-y^2}},$$

$$\ddot{y} + \frac{y\,\dot{y}^2}{1-y^2} + y\,(1-y^2)\dot{\gamma}^2 = y\sqrt{1-y^2}\,(\ddot{x}_0\,\sin\gamma - \cos\gamma).$$

The length l of an equivalent spherical pendulum is found by equating the natural frequency of the pendulum to that of the first antisymmetric mode ω_{11} [cf. Eq. (1)], for liquid sloshing

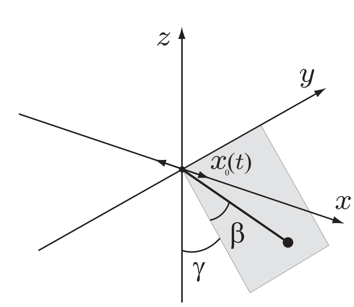

FIG. 6. Coordinate system for the spherical pendulum model: the pendulum is shown in the shaded plane and driven horizontally in the x direction with time-dependent forcing $x_0(t)$.

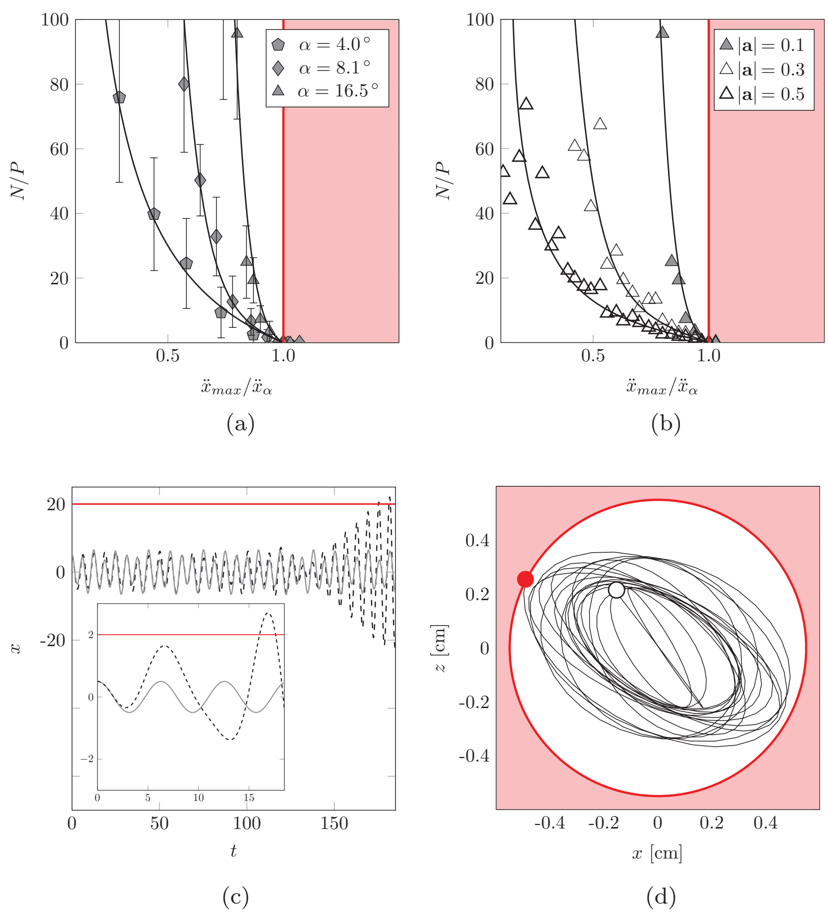

FIG. 7. (Color online) Modeling the sloshing of a liquid in a cup as an equivalent spherical pendulum. (a) shows steps to spill N/P versus the normalized initial deflection of the interface $\ddot{x}_{\max}/\ddot{x}_{\alpha}$ for three different coffee levels ($8° \lesssim \alpha \lesssim 16°$). Each data point shown in the figure corresponds to the average number of steps N of all model runs that yield a spill divided by the fraction of runs P that result in a spill. The error bars suggest the scatter in the number of steps for any given run for any fixed conditions due to the random nature of the noise in the forcing signal; solid lines through data sets are meant to guide the eye. (b) reflects the influence of the noise amplitude $|\mathbf{a}|$ on the number of steps to spill for $\alpha = 16.5°$. Each data point here corresponds to an average as discussed in the previous figure, but the error bars have been left out for clarity. (c) shows the effect of a time-dependent (chirped) forcing frequency on coffee spilling over short (inset) and longer times. (d) shows an example trajectory of a spherical pendulum projected onto the xz plane in the case $\alpha = 16.5°$. The starting position of the pendulum is denoted by an open circle and is based on the initial angle in the xy plane and a random z component. The "critical spill radius" in (d) is denoted by the red circle.

[8]. As the idea was to develop as simple a model as possible, the forcing in the x direction includes the walking frequency ω_{cup} with amplitude A similar to that observed in experiments *and* a number of higher-frequency harmonics [30]:

$$x_0(t) = A \sin \omega_{\mathrm{cup}} t + \sum_{n=1}^{n^*} a_n \sin(\omega_n t + \phi_n), \qquad (4)$$

with randomly selected frequencies ω_n over a sufficiently large interval, which includes ω_{cup}, as well as random phase shifts ϕ_n and amplitudes a_n, so that their total amplitude in the Euclidean norm $|\mathbf{a}| = \sqrt{\sum_n a_n^2}$ is a fraction of A to reflect the presence of the noise [see Fig. 3(c)]. The number of harmonics n^* is taken sufficiently large so that the results in Fig. 7 become insensitive to further increases in n^*.

The spherical pendulum model was integrated from random initial conditions and with randomly selected forcing (4). A considerable number of runs was performed to collect statistical data of spilling events, which occur when the deflection of the pendulum exceeds a critical value corresponding to the spill angle α [see Fig. 2(c)]. Qualitative agreement exists between the outcome of these simulations and the presented experimental observations, namely, that with increasing $\ddot{x}_{\max}/\ddot{x}_{\alpha}$ the number of steps to spill, normalized by the fraction of runs P that result in a spill, N/P, decreases significantly up to $\ddot{x}_{\max}/\ddot{x}_{\alpha} = 1$, at which point there is a nearly instantaneous spill. Furthermore, N/P decreases with an increase in the coffee level (decrease in α) for any value of $\ddot{x}_{\max}/\ddot{x}_{\alpha}$. All data points in Fig. 7(a), shown in gray, correspond to the low value of the noise amplitude used, $|\mathbf{a}|/A = 0.1$, which is analogous to the focused walking characterized by lower noise levels (gray symbols) in Fig. 7(b). The effect of increasing the amplitude $|\mathbf{a}|$ of the noise in the forcing signal is to decrease N/P as shown in Fig. 7(b). This is qualitatively similar in nature to the differences observed in experiments between focused and unfocused walking [see Fig. 5].

Next, since walking starts from rest and is usually nonuniform, the frequency of step changes so that one can expect resonance to occur much earlier than when excitation approaches the natural frequency [31]. This is known as the *chirped* (jump [31]) frequency phenomenon [32] used to trap particles with electromagnetic waves—resonance sets in when the ramped forcing (chirped) frequency is midway between its initial value and the natural frequency for resonance in the unforced problem. Therefore, since walking is usually nonuniform, the step frequency varies initially from zero to some constant value (typically ∼0.6 of the natural frequency) and over the course of walking, one can expect that resonance will occur even before the step frequency reaches the value of the natural frequency. If this is true, then perhaps the liquid in the cup experiences growth in sloshing at angular frequencies ω lower than the natural frequency ω_{nat} and could lead to spilling. This behavior is exemplified in Fig. 7(c), in which a simple planar pendulum is subject to a ramped forcing frequency ω starting from zero with an arbitrary deflection at $t = 0$ [shown as the black (dashed) line]. For the particular case shown in Fig. 7(c), $x_0 = 5$ and the ramp rate $\epsilon = 0.003\,18$ results in a frequency ratio $\omega/\omega_{\rm nat} = 0.6$ after 30 cycles. At some point in time, it is observed that the pendulum experiences an increase in the x deflection corresponding to the frequency reaching the "jump frequency." If the maximum experienced amplitude is above a critical deflection amplitude [shown as the red (solid horizontal) line], then spilling occurs. This should be contrasted to a simple planar pendulum with a fixed forced frequency [shown as the gray (solid) line] corresponding to a fixed frequency ratio 0.6, in which case no substantial increase in the x deflection is achieved. The inset of Fig. 7(c) corresponds to the ramp rate $\epsilon = 0.0318$, but $\omega/\omega_{\rm nat} = 0.6$ is reached after three cycles to model the initial stage of walking from rest. It is apparent that the deflection x increases at early times and can reach a critical amplitude versus the motion of an unforced pendulum with the same initial deflection.

A closer look at the liquid motion in the cup suggests that not only back-and-forth but also swirling liquid motions can be excited by walking at typical speeds [see Fig. 1(b)]. While swirl does not contribute much to coffee spilling, it is still an interesting question as to why rotational motion is generated. In addition to the obvious potential reason—movement of the hand in the xz plane—many different experimental studies concerned with back-and-forth sloshing have revealed an interesting type of liquid instability occurring very close to the lowest liquid resonant frequency [8]: the nodal diameter of the antisymmetric wave begins to rotate at an unsteady rate. The essential features of this complex liquid motion can be described qualitatively as an apparent "rotation" of the liquid about the vertical axis of symmetry of the tank, superimposed on the normal sloshing motion. There is also a somewhat larger frequency range just above the natural frequency in which the nodal diameter can rotate stably at a constant rate [10]. As illustrated in Fig. 7(d), the spherical pendulum does demonstrate, depending on the ratio of the excitation to the natural frequency, instability of planar motion and thus generation of rotational motion.

III. DISCUSSION

As follows from the presented study, we spill coffee either by accelerating too much for a given coffee level (fluid statics) or through more complicated dynamical phenomena due to the *particular* range of sizes of common coffee cups, which is dictated by the convenience of carrying them and the normal consumption of coffee by humans. Namely, first the maximum acceleration occurring early on in the walking sets an initial sloshing amplitude. This interface deflection is then amplified by the back-and-forth and pitching excitations. Vertical excitation does not lead to resonance as it is a subharmonic excitation (Faraday phenomenon). The noise component of motion contains higher-frequency harmonics, which make the antisymmetric mode unstable, thus generating a swirl. Time to spill generally depends on whether walking is in a focused or unfocused regime and increases with decreasing maximum acceleration (walking speed). The difference between the focused and unfocused regimes suggests that walking with coffee is a control problem, since the number of steps to spill is different in these two regimes. As the data analysis shows, this is due to the difference in the level of a noise excitation, while the amplitude of a regular excitation does not change significantly, which is conceivably due to the natural constraints of the biomechanics of walking. However, whether it is a feedback (closed loop) control system—i.e., when a human being "identifies" the resonant sloshing frequency and then performs a targeted suppression of the resonant mode—or an open loop control system—i.e., when a human being simply becomes more careful about carrying a cup regardless of the natural frequency of the fluid in it—may depend on an individual. Understanding this is beyond the scope of our work, as the collected data are too limited to provide conclusive results about such a specific question as to whether it is only the resonant excitation which is suppressed in the focused regime. Another relevant question to study is the influence of the degree of danger of the fluid in a cup, e.g., hot vs cold.

Our simple theoretical model captures the observed phenomena qualitatively—quantitative differences are in part due to differences in the widths of the resonance curves between spherical pendulum and liquid oscillations in a cylindrical container; in the latter case, the response curve is substantially wider (cf. Fig. 2.4 in [10]), thus explaining the stronger response of the liquid to excitations at frequencies far from resonance. The fact that the studied system demonstrates that in the focused regime, characterized by lower levels of noise, the coffee spill occurs later compared to the unfocused regime makes the phenomenon analogous [33] to that of stochastic resonance [34–36], where the periodic forcing is amplified by the presence of noise above a certain threshold.

While the experimental data discussed here are representative of human walking with a cup of coffee, as the involved parameters—excitation amplitudes and frequencies [2,3]—do not have much variation among individuals, future comparative studies may provide further insights into the variability of the discussed phenomena. For example, gender differences [5]—gait velocity and step length are lower and step frequency is higher for women than for men—may lead to differences in coffee spilling. However, some age effects [5], such as the changes of the basic gait parameters most frequently seen with advancing age, are a reduction of gait speed and step length, but just small changes of the step frequency are not expected to lead to variations in coffee spilling.

Finally, lessons learned from sloshing engineering may suggest strategies to control spilling, e.g., via using (a) a flexible container to act as a sloshing absorber in suppressing liquid oscillations, or (b) a series of concentric rings (baffles) arranged around the inner wall of a container [11,12]. Namely, ring baffles damp the large sloshing mass flow associated with the main resonant frequency of the liquid in the container into several smaller sloshing masses—high-speed turbulent flow around baffles—having considerably higher first-mode resonant frequencies [11]. The baffles can be nonperforated or perforated. Perforation provides better damping and reduces the mass of baffles. However, despite the variety of spill control options, the simplicity and convenience of a common coffee cup [see Fig. 1(a)] will likely continue to outweigh the side effect of coffee drinking studied here—occasional spilling.

ACKNOWLEDGMENTS

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[13] Overground experiments were implemented. The reason to avoid treadmill experiments is because the start of the motion should be natural: it is known from biomechanics that treadmill experiments introduce some limitations [6] and it could be the case that the legs are being driven by the treadmill and the arms are working out a phase relationship that is compatible with their resonant frequency.
[14] In both cases though, the subject was aware of the fact that the cup was nearly full of coffee.
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